Exercise 38

In a fish farm, a population of fish is introduced into a pond and harvested regularly. A model for the rate of change of the fish population is given by the equation

$$\frac{dP}{dt} = r_0\left(1 - \frac{P(t)}{P_c}\right)P(t) - \beta P(t)$$

where r_0 is the birth rate of the fish, P_c is the maximum population that the pond can sustain (called the *carrying capacity*), and β is the percentage of the population that is harvested.

(a) What value of dP/dt corresponds to a stable population?

(b) If the pond can sustain 10,000 fish, the birth rate is 5%, and the harvesting rate is 4%, find the stable population level.

(c) What happens if β is raised to 5%?

Solution

Part (a)

Since dP/dt represents the rate of population growth as t increases, a stable population is one for which

$$\frac{dP}{dt} = 0.$$

Part (b)

To find the stable population level, set $dP/dt = 0$, $P_c = 10{,}000$, $r_0 = 0.05$, and $\beta = 0.04$.

$$\frac{dP}{dt} = r_0\left(1 - \frac{P(t)}{P_c}\right)P(t) - \beta P(t)$$

$$0 = 0.05\left(1 - \frac{P(t)}{10\,000}\right)P(t) - 0.04P(t)$$

Solve for $P(t)$.

$$0 = 0.05P(t) - 0.000005[P(t)]^2 - 0.04P(t)$$

$$0 = 0.01P(t) - 0.000005[P(t)]^2$$

$$0 = P(t)[0.01 - 0.000005P(t)]$$

$$P(t) = 0 \quad \text{or} \quad 0.01 - 0.000005P(t) = 0$$

$$P(t) = 0 \quad \text{or} \quad P(t) = 2000$$

The stable fish population in this case is 2000.

Part (c)

To find the stable population level, set $dP/dt = 0$, $P_c = 10{,}000$, $r_0 = 0.05$, and $\beta = 0.05$.

$$\frac{dP}{dt} = r_0\left(1 - \frac{P(t)}{P_c}\right)P(t) - \beta P(t)$$

$$0 = 0.05\left(1 - \frac{P(t)}{10\,000}\right)P(t) - 0.05P(t)$$

Solve for $P(t)$.

$$0 = 0.05P(t) - 0.000005[P(t)]^2 - 0.05P(t)$$

$$0 = -0.000005[P(t)]^2$$

$$0 = [P(t)]^2$$

$$P(t) = 0$$

If β is raised to 5%, there won't be a stable fish population.